10. A carrier wave is frequency-modulated using a sinusoidal signal of frequency f_m and amplitude A_m.
   a) If $f_m = 1\ KHz$ and A_m is increased from 0 volts, it is found that the carrier frequency of the FM signal is reduced to zero for the **first** time when $A_m = 2\ volts$. What is the frequency sensitivity, k_f of the modulator?
   b) What is the value of A_m for which the carrier component is reduced to zero for the **second** time?

Solution:

a) $b = \dfrac{\Delta f}{f_m} = \dfrac{k_f A_m}{f_m} \quad \Rightarrow \quad k_f = \dfrac{b\, f_m}{A_m}$

Since $J_0(b) = 0$ for the first time is when $b \approx 2.44$,

Thus $\quad k_f = \dfrac{2.44 \times 10^3}{2} = 1.22\ KHz / volt.$

b) $J_0(b) = 0$ for the second time is when $b \approx 5.52$, with $k_f = 1.22\ KHz / volt.$

Thus, $\quad k_f = 1.22\ KHz / volt. = \dfrac{5.52 \times 10^3}{A_m} \quad \Rightarrow \quad A_m = \dfrac{5.52 \times 10^3}{1.22 \times 10^3} = 4.52\ volts$

11. An FM signal with a frequency deviation of 10 KHz at a modulation frequency of 5 KHz is applied to two frequency multipliers connected in series. The **first** multiplier **doubles** the frequency and the **second** multiplier **triples** the frequency.
   a) Determine the **frequency deviation** and the **modulation index** of the FM signal obtained at the **second** multiplier output.
   b) What is the **frequency separation** between adjacent sidebands of this FM signal obtained at the **second** multiplier output?

Solution:

a) Overall frequency multiplication: $\quad n = 2 \times 3 = 6$

Assume the instantaneous frequency of the FM signal at the input of the first frequency multiplier is $\quad f_{i_1}(t) = f_c + \Delta f \cos(2p\, f_m\, t)$

Then the instantaneous frequency of the resulting FM wave at the output of the second frequency multiplier is $\quad f_{out_2}(t) = n f_c + n \Delta f \cos(2p\, f_m\, t)$

The overall frequency deviation $\quad = n \Delta f = 6 \times 10 = 60$

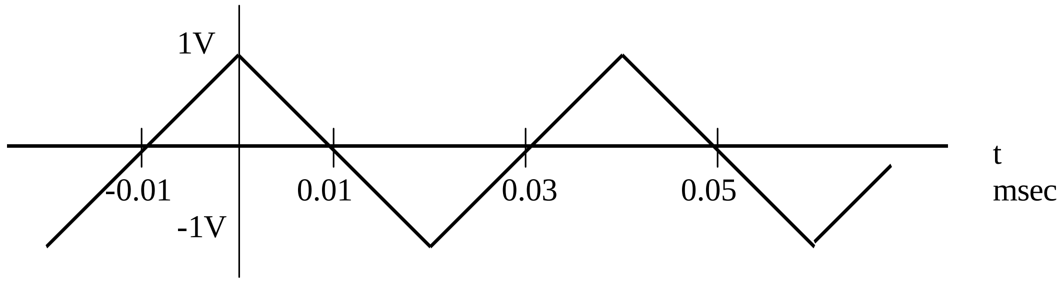
Since $f_m = 5\ KHz \quad \Rightarrow \quad b = \dfrac{n \Delta f}{f_m} = \dfrac{60}{5} = 12$

b) The frequency separation of the adjacent side frequencies of this FM signal = $f_m = 5\ KHz$

12. A triangular wave modulating signal is as shown below.

a) If it is **phase modulated**, locate and label on the modulating signal where the *highest* and the *lowest* frequency will occur in the phase modulated signal.

b) Determine the **highest** and the **lowest** frequency if the carrier frequency is at 100 MHz and $k_p = 4\ Hz/volt. = 8\pi\ radian/volt.$



Solution:

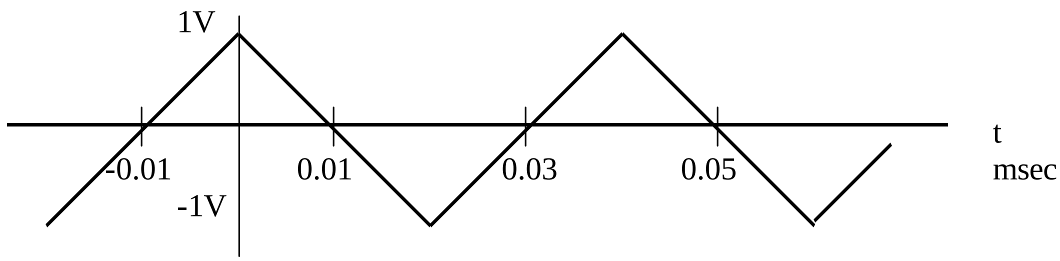
$$\theta_i(t) = 2\pi f_c t + k_p m(t) = 2\times10^8 \pi t + 8\pi f_\Delta(t) \quad \Rightarrow \quad \omega = \frac{d\theta}{dt} = 2\times10^8\pi + 8\pi \frac{d f_\Delta}{dt}$$

$$f = \frac{1}{2\pi}\frac{d\theta}{dt} = 10^8 + 4\frac{d f_\Delta}{dt} \quad where \quad \frac{d f_\Delta}{dt} = \pm\frac{2V.}{0.02\times10^{-3}} = \pm10^5\ V/\text{sec}.$$

$$f_{\max} = (100 + 4\times10^5)\ MHz = 100.4\ MHz \quad ; \quad f_{\min} = (100 - 4\times10^5)\ MHz = 99.6\ MHz$$

c) If it is **frequency modulated**, locate and label on the modulating signal where the *highest* and the *lowest* frequency will occur in the frequency modulated signal.

d) Determine the **highest** and the **lowest** frequency if the carrier frequency is at 100 MHz and $k_f = 4\ KHz/volt. = 8\pi\times10^3\ radian/volt.$



Solution:

$$f_i(t) = f_c + k_f m(t) = 10^8 + 4\times10^3 f_\Delta(t)$$

$$f_{\max} = 100\ MHz + 4\ KHz = 100.004\ MHz \quad ; \quad f_{\min} = 100\ MHz - 4\ KHz = 99.996\ MHz$$

13. A carrier wave of frequency 100 MHz is frequency-modulated by a sinusoidal wave of amplitude 20 volts and frequency 100 KHz. The frequency sensitivity of the modulator is 25 KHz per volt. Determine the approximate bandwidth of the FM signal by using Carson's rule.

Solution:

$\Delta f = k_f A_m = 25\times 10^3\ Hz / volt \times 20\, volt. = 5\times 10^5\ Hz$

$b = \dfrac{\Delta f}{f_m} = \dfrac{5\times 10^5}{10^5} = 5$

Using Carson's Rule: Bandwidth $= 2 f_m (1+b) = 2\times 10^5 \times 6 = 1.2\ MHz$

14. Commercial FM band is from 88~108 MHz. Each FM station is assigned at 200 KHz apart and the peak frequency deviation is set at 75 KHz. **Determine** the followings:

a) Determine the highest frequency that is allowed in the message or information signal?
b) What is the maximum number of FM stations can exist within the FM band?
c) Determine the bandwidth of the FM signal, if the highest frequency in the information signal is 15 KHz.
d) From the provided information in this problem, is the commercial FM transmission in narrowband FM or wideband FM format? Explain why?

Solution:

a) Carson's rule,
$B \approx 2(\Delta f + f_m) = 2(75 + f_m) = 200\ KHz \Rightarrow 2 f_m = 50\ KHz \Rightarrow f_m = 25\, KHz$

b) 108-88=20 MHz
Frequency apart between two consecutive FM stations = 200 KHz
\# of FM stations = $\dfrac{20000}{200} = 100$

c) If $f_m = 15\, KHz$, $\Delta f = 75\, KHz$, $\Rightarrow B = 2(75+15) = 180\, KHz$

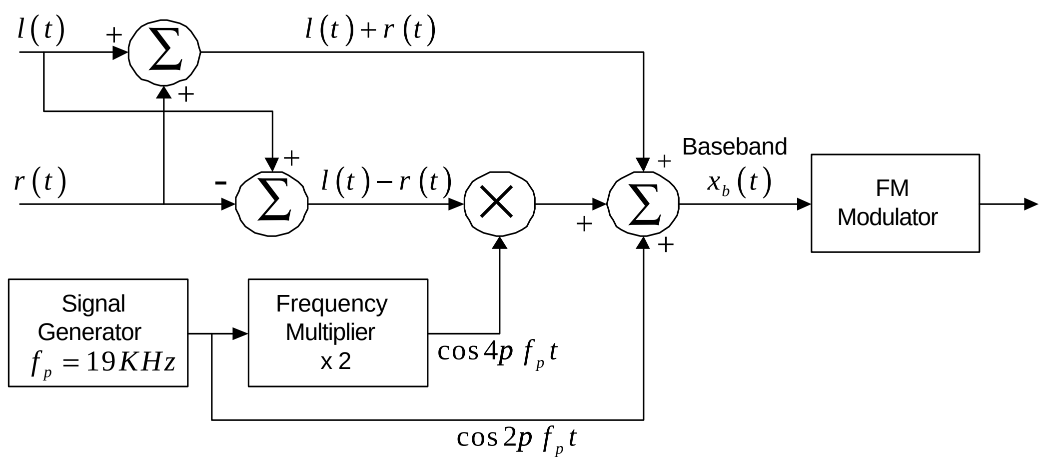
d) Since $b = \dfrac{\Delta f}{f_m} = \dfrac{75}{15} = 5 \Rightarrow WBFM$

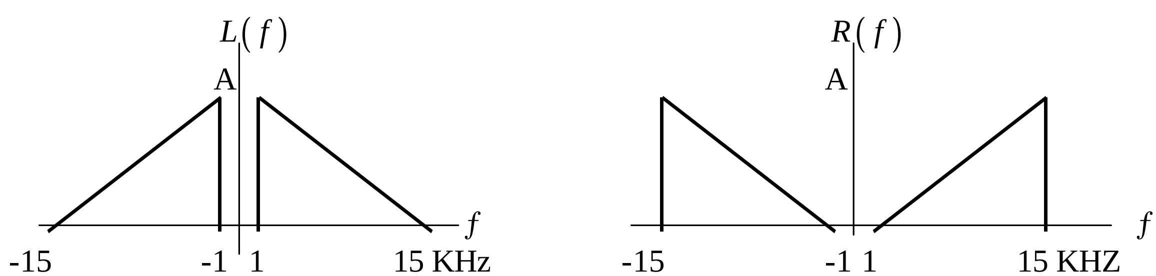
15. FM stereo broadcasting is accomplished by using DSB modulation for multiplexing and FM modulation for transmission. In the system illustrated below, $l(t)$ and $r(t)$ are the left and right channel signals respectively. Assume that both the left and right channel is bandlimited up to 15 KHz as shown in $L(f)$ and $R(f)$ in frequency domain.

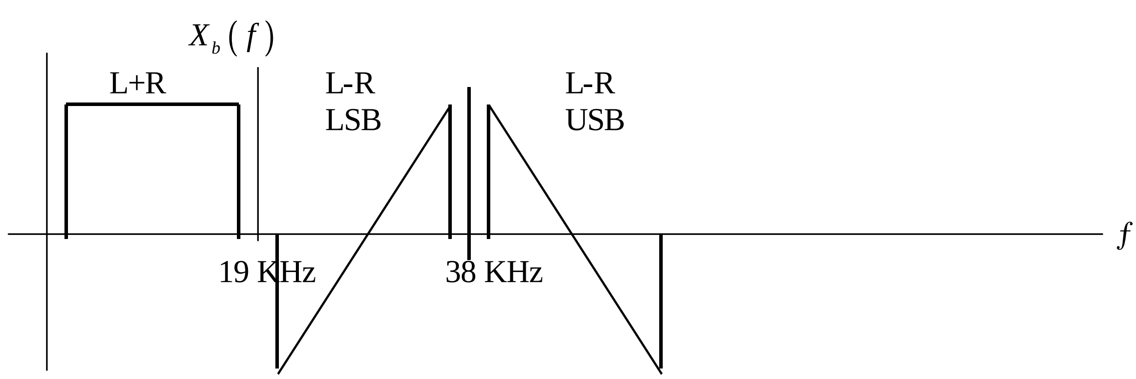
a) What frequency is set for the pilot-carrier that is used as a stereo broadcasting indicator for the stereo receiver?

Pilot carrier for the FM stereo broadcasting is set at ____19____ KHz.

b) **Sketch** $X_b(f)$ at the FM modulator input, and **label** all particular points.







16. A FM signal is expressed as the following:

$$f_{FM}(t) = 100\cos(2\pi f_c t + 0.8\sin 2\pi f_m t + 1.6\sin 4\pi f_m t) \quad \text{where } f_c = 100\ MHz \text{ and } f_m = 1\ KHz$$

a) Determine the instantaneous frequency of $f_{FM}(t)$ at $t = 0$.

b) Determine the peak frequency deviation of $f_{FM}(t)$.

Solution:

a) $f_{FM}(t) = 100\cos(2\pi f_c t + 0.8\sin 2\pi f_m t + 1.6\sin 4\pi f_m t)$

$$\theta(t) = 2\pi f_c t + 0.8\sin 2\pi f_m t + 1.6\sin 4\pi f_m t$$

$$\frac{d\theta}{dt} = 2\pi f_c + (2\pi f_m)\times 0.8\cos 2\pi f_m t + (4\pi f_m)\times 1.6\cos 4\pi f_m t$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta}{dt} = f_c + 0.8 f_m \cos 2\pi f_m t + 3.2 f_m \cos 4\pi f_m t$$

$$f_i(t)\big|_{t=0} = f_c + 0.8 f_m + 3.2 f_m = 100\,MHz + 0.8\,KHz + 3.2\,KHz = 100.004\,MHz$$

b) Since $f_{FM}(t) = 100\cos(2\pi f_c t + 0.8\sin 2\pi f_m t + 1.6\sin 4\pi f_m t)$

and the range of cosine function is from –1 to +1.

Therefore the peak frequency deviation of $f_{FM}(t)$ is at the maximum at $t=0$.

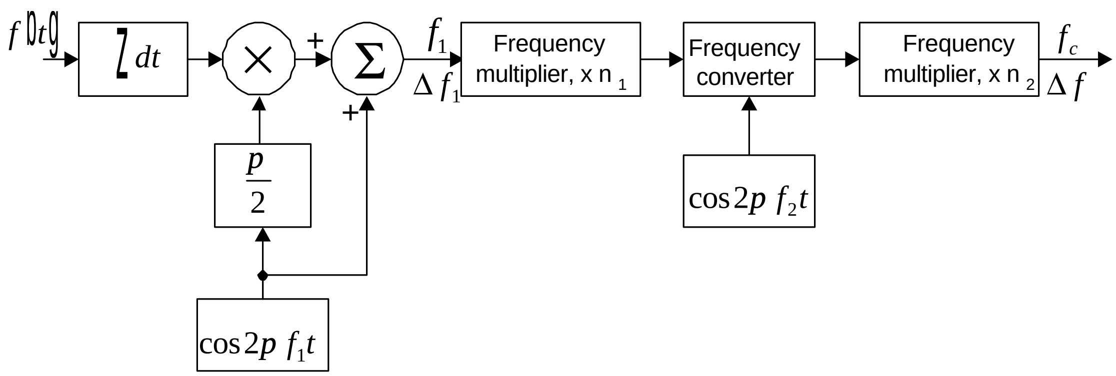
The peak frequency deviation of $f_{FM}(t)$ = (0.8+3.2) KHz =4 KHz

17. Determine the followings for the output of the FM transmitter as shown below where,

$$f_1 = 200\,KHz; \quad f_2 = 10.8\ MHz; \quad \Delta f_1 = 25\,Hz; \quad n_1 = 64 \ \& \quad n_2 = 48$$

a) The carrier frequency f_c at the output of the FM transmitter.

b) The peak frequency deviation Δf at the output of the FM transmitter.

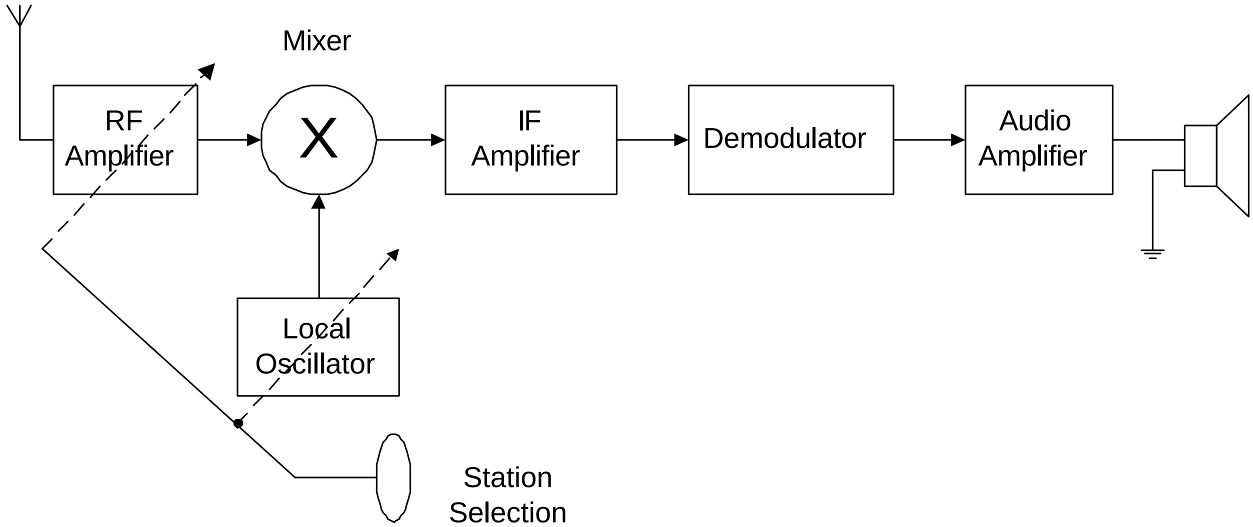


Solution:

a) $f_c = (0.2\,MHz\times 64 \pm 10.8)\times 48 = 96\,MHz \quad or \quad 1132.8\,MHz$

b) $\Delta f = 25\times 64\times 48 = 76.8\,KHz$

18. The schematic diagram of a superheterodyne receiver is as shown below:



Intermediate Frequency is set at 455 KHz, and the AM signal has a bandwidth of 10 KHz.

The AM frequency band is from ________540___ KHz to ______1600________ KHz.

Frequency range of the local oscillator is from _________995___ KHz to _____2055__ KHz.

If the local oscillator is at 1355 KHz, the carrier freq, f_c =_____1355–455=900_______ KHz.

If the local oscillator is at 1355 KHz, the image frequency =____900+2(455)=1810____ KHz.

The bandwidth of the IF Amplifier is from___455–5=450____ KHz to ___455+5=460___KHz.

Describe the advantages of the superheterodyne receiver.

Do without a tunable filter.
It is simple and easy to operate